Utility Analysis

Part of Ecological Network Analysis

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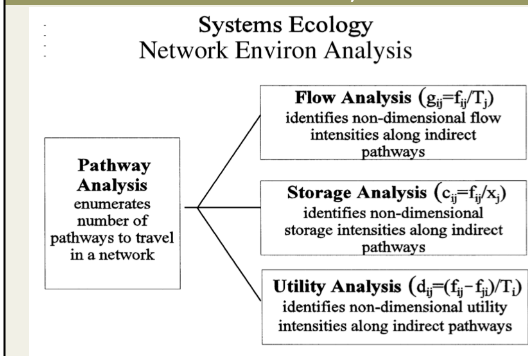
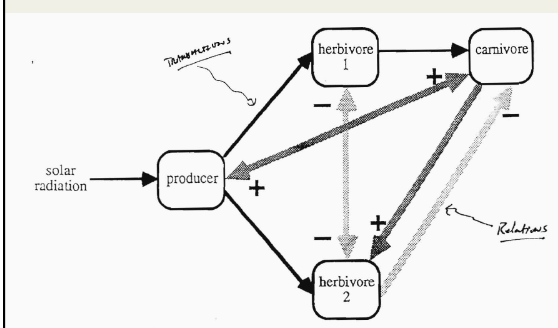
Network Environ Analysis Overview



Fath and Patten 1999

Transactions & Relations



Coaction Theory

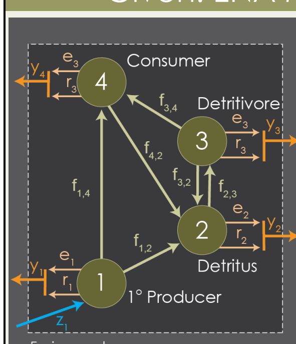
Nine possible qualitative relationships from (+,0,-)
(After Burkholder 1954)

	+	0	-
+	(+,+) mutualism	(+,-) commensalism	(+,-) exploitation (predation)
0	(0,+) Commensal host	(0,0) neutralism	(0,-) amensalism
-	(-,+) exploited (prey)	(-,0) amensal host	(-,-) competition

as in Fath 2007

Utility Calculations

Given: ENA Model Data



- Node Names
- F: Flows from *i* to *j*
- z: Inputs
- e: Exports
- r: Respirations
- y: Outputs (e + r)
- Living: logical (T or F)

Utility Analysis Algebra - Flow

Given F, T, G, and G' as defined in flow analysis

Net intensive direct flow matrix

$$\mathbf{D} = [d_{ij}] = (f_{ij} - f_{ji})/T_i$$

$$\mathbf{D} = G' - G^T \qquad -1 \leq d_{ij} \leq 1$$

$$\mathbf{sD} = (sd_{ij}) = \text{sign of } d_{ij}$$

$$(sd_{i,j} \rightarrow sd_{j,i}) \qquad (sd_{i,j}, sd_{j,i})$$

Note *row to column* orientation — Local signed direct relationship

Net intensive integral flow matrix

Must check if matrix is convergent

$$\mathbf{U} = (u_{ij}) = \sum_{m=0}^{\infty} (D)^m \qquad -\infty \leq (u_{ij}) \leq \infty$$

$$\mathbf{sU} = (sd_{ij}) = \text{sign of } u_{ij} \qquad (su_{i,j}, su_{j,i})$$

Global (net) relationships

Absolute or Realized Utilities - Flow

$$\Delta = \text{diag}(\vec{T}) \cdot \mathbf{D}$$ Direct or Local

$$\Upsilon = \text{diag}(\vec{T}) \cdot \mathbf{U}$$ Integral or Global

Utilities are rescaled by throughflow vector

Synergism and Mutualism Statistics

Network Synergism

Fath 1998

$$\frac{\sum(\Upsilon)_+}{\sum(\Upsilon)_-}$$

Sum of the positive integral utility values
Divided by the sum of the negative values

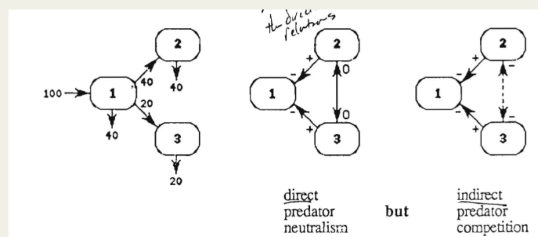
Network Mutualism

Fath 2007

$$\frac{\sum(sign(\Upsilon))_+}{\sum(sign(\Upsilon))_-}$$

Count of positive utility elements
Divided by the count of the negative utilities

Utility Example – 2 predators, 1 prey



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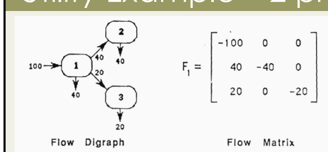
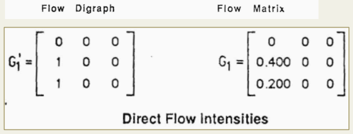
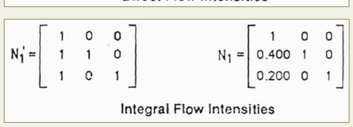
Utility Example – 2 predators, 1 prey







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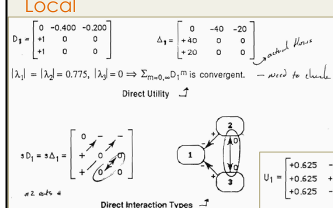
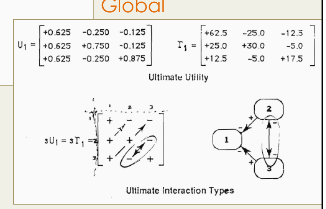
Utility Example – 2 predators, 1 prey

Local



Global



Interaction Type Summary

Local	Global
# + = 2	# + = 5
# - = 2	# - = 4
# 0 = 2	# 0 = 0
+/- = 1	+/- = 1.25

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Utility Example – 2 predators, 1 prey

	Direct (Δ_1 matrix) (utiles)	Ultimate (Υ_1 Matrix) (flux-utiles)
Utility Summary		
Sum of + utilities	+60	+147.5
Sum of – utilities	–60	–47.5
|Benefit (+)/cost (–)| ratio	1.00	3.11
Interaction Type Summary		
Number of + signs	2	5
Number of – signs	2	4
Number of +/number of – signs	1.00	1.25

Network Synergism
Robust

Network Mutualism
Weak measure

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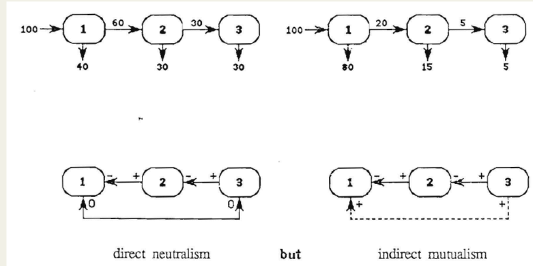
Utility Example – Food Chain



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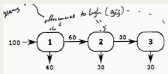
Utility Example – Food Chain



Flow Digraph

$$F_{3a} = \begin{bmatrix} -100 & 0 & 0 \\ 60 & -60 & 0 \\ 0 & 30 & -30 \end{bmatrix}$$

Flow Matrix

$$G'_{3a} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad G_{3a} = \begin{bmatrix} 0 & 0 & 0 \\ 0.600 & 0 & 0 \\ 0 & 0.500 & 0 \end{bmatrix}$$

Direct Flow Intensities

$$N'_{3a} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad N_{3a} = \begin{bmatrix} 1 & 0 & 0 \\ 0.600 & 1 & 0 \\ 0.300 & 0.500 & 1 \end{bmatrix}$$

Integral Flow Intensities

$$D_{3a} = \begin{bmatrix} 0 & -0.600 & 0 \\ +1 & 0 & -0.500 \\ 0 & +1 & 0 \end{bmatrix}$$

$|\lambda_1| = |\lambda_2| = 1.049,\ |\lambda_3| = 0 \Rightarrow \Sigma_{m=0,\infty} D^m$ is divergent.

Direct Utility

Analysis Fails
Divergent Series

Network Utility, Rule 1 (?)

Food chains with high transfer efficiencies cannot produce utility

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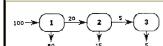
Utility Example – Food Chain



Flow Digraph

$$F_{3b} = \begin{bmatrix} -100 & 0 & 0 \\ 20 & -20 & 0 \\ 0 & 5 & -5 \end{bmatrix}$$

Flow Matrix

$$G'_{3b} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad G_{3b} = \begin{bmatrix} 0 & 0 & 0 \\ 0.200 & 0 & 0 \\ 0 & 0.250 & 0 \end{bmatrix}$$

Direct Flow Intensities

$$N'_{3b} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad N_{3b} = \begin{bmatrix} 1 & 0 & 0 \\ 0.200 & 1 & 0 \\ 0.050 & 0.250 & 1 \end{bmatrix}$$

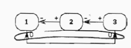
Integral Flow Intensities

$$D'_{3b} = \begin{bmatrix} 0 & -0.200 & 0 \\ +1 & 0 & -0.250 \\ 0 & +1 & 0 \end{bmatrix} \quad \Delta_{3b} = \begin{bmatrix} 0 & -20 & 0 \\ +20 & 0 & -5 \\ 0 & +5 & 0 \end{bmatrix}$$

$|\lambda_1| = |\lambda_2| = 0.652,\ |\lambda_3| = 0 \Rightarrow \Sigma_{m=0,\infty} D_{3b}^m$ is convergent.

Direct Utility

$$sD_{3b} = s\Delta_{3b} = \begin{bmatrix} 0 & - & 0 \\ + & 0 & - \\ 0 & + & 0 \end{bmatrix}$$

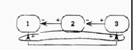


Direct Interaction Types

$$U_{3b} = \begin{bmatrix} +0.862 & -0.138 & +0.035 \\ +0.690 & +0.690 & -0.172 \\ +0.690 & +0.690 & +0.828 \end{bmatrix} \quad \Upsilon_{3b} = \begin{bmatrix} +86.2 & -13.8 & +3.5 \\ +13.8 & +13.8 & -3.5 \\ +3.5 & +3.5 & +4.1 \end{bmatrix}$$

Ultimate Utility

$$sU_{3b} = s\Upsilon_{3b} = \begin{bmatrix} + & - & + \\ + & + & - \\ + & + & + \end{bmatrix}$$



Ultimate Interaction Types

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Utility Example – Food Chain

	Direct (Δ_{3b} matrix) (utiles)	Ultimate (Υ_{3b} Matrix) (flux-utiles)
Utility Summary		
Sum of + utilities	+25	+128.1
Sum of – utilities	–25	–17.3
|Benefit (+)/cost (–)| ratio	1.00	7.40
Interaction Type Summary		
Number of + signs	2	7
Number of – signs	2	2
Number of +/number of – signs	1.00	3.50

Network Synergism and Mutualism tell us that network interactions add to the system – more positive overall.

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Utility Example – Oyster Reef Model

Direct Utility

$$D_4 = \begin{bmatrix} 0 & -0.381 & 0 & 0 & 0 & -0.012 \\ +0.709 & 0 & -0.360 & -0.136 & +0.057 & +0.015 \\ 0 & +1.000 & 0 & -0.148 & -0.148 & 0 \\ 0 & +0.358 & +0.142 & 0 & -0.078 & 0 \\ 0 & -0.504 & +0.481 & +0.263 & 0 & -0.069 \\ +0.749 & -0.475 & 0 & 0 & +0.251 & 0 \end{bmatrix} \quad \Delta_4 = \begin{bmatrix} 0 & -15.79 & 0 & 0 & 0 & -0.514 \\ +15.79 & 0 & -8.172 & -3.035 & +1.465 & +0.326 \\ 0 & +8.172 & 0 & -1.206 & -1.206 & 0 \\ 0 & +3.035 & +1.206 & 0 & -0.661 & 0 \\ 0 & -1.465 & +1.206 & +0.661 & 0 & -0.172 \\ +0.514 & -0.326 & 0 & 0 & +0.172 & 0 \end{bmatrix}$$

$|\lambda_1| = |\lambda_2| = 0.899,\ |\lambda_3| = |\lambda_4| = 0.222,\ |\lambda_5| = |\lambda_6| = 0.052 \Rightarrow \Sigma_{m=0,\infty} D_4^m$ is convergent.

Direct Utility

$$sD_4 = s\Delta_4 = \begin{bmatrix} 00 & - & 0 & 0 & 0 & - \\ + & 00 & - & - & + & + \\ 0 & + & 00 & - & - & 0 \\ 0 & + & + & 00 & - & 0 \\ 0 & - & + & + & 00 & - \\ + & - & 0 & 0 & + & 00 \end{bmatrix} \Rightarrow \begin{bmatrix} N & & & & & \\ P & N & & & & \\ N & P & N & & & \\ N & P & P & N & & \\ N & L & P & P & N & \\ P & L & N & N & P & N \end{bmatrix}$$

N = neutralism (0,0)
P = nihilism (predation) (+,–)
L = altruism (–,+)

Direct Interaction Types

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Utility Example – Oyster Reef Model

Integral Utility

$$U_4 = \begin{bmatrix} +0.833 & -0.223 & +0.071 & +0.013 & -0.027 & -0.012 \\ +0.424 & +0.599 & -0.194 & -0.036 & +0.065 & -0.001 \\ +0.394 & +0.547 & +0.741 & -0.200 & -0.061 & +0.007 \\ +0.208 & +0.287 & +0.002 & +0.946 & -0.056 & +0.005 \\ +0.001 & +0.066 & +0.437 & +0.166 & +0.911 & -0.061 \\ +0.423 & -0.435 & +0.255 & +0.068 & +0.177 & +0.976 \end{bmatrix}$$

$$\Upsilon_4 = \begin{bmatrix} +34.553 & -9.240 & +2.928 & +0.531 & -1.197 & -0.485 \\ +9.440 & +13.333 & -4.311 & -0.799 & +1.450 & -0.022 \\ +3.127 & +4.472 & +6.059 & -1.635 & -0.498 & +0.060 \\ +1.762 & +2.435 & +0.017 & +8.021 & -0.478 & +0.047 \\ +0.003 & +0.116 & +1.097 & +0.418 & +2.286 & -0.154 \\ +0.290 & -0.299 & +0.175 & +0.047 & +0.122 & +0.670 \end{bmatrix}$$

Ultimate Utility

$$sU_4 = s\Upsilon_4 = \begin{bmatrix} ++ & - & + & + & - & - \\ + & ++ & - & - & + & - \\ + & + & ++ & - & - & + \\ + & + & + & ++ & - & + \\ + & + & + & + & ++ & - \\ + & - & + & + & + & ++ \end{bmatrix} \Rightarrow \begin{bmatrix} M & & & & & \\ P & M & & & & \\ M & P & M & & & \\ M & P & P & M & & \\ P & M & P & P & M & \\ P & K & M & M & P & M \end{bmatrix}$$

M = mutualism (+,+)

K = competition (−,−)

Ultimate Interaction Types

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Utility Example – Oyster Reef Model

	Direct (Δ_4 matrix) (utiles)	Ultimate (Υ_4 Matrix) (kcal m^{-2} d^{-1}-utiles)
Utility Summary		
Sum of + utilities	+32.547	+93.459
Sum of − utilities	−32.547	−19.118
|Benefit (+)/cost (−)| ratio	1.00	4.89 → Synergism
Interaction Type Summary		
Number of + signs	10	25
Number of − signs	10	11
Number of +/number of − signs	1.00	2.17 → mutualism

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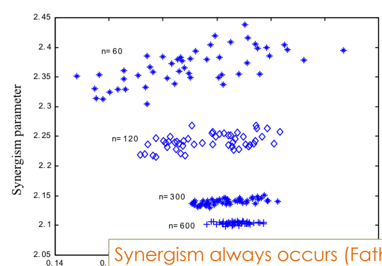
Generality of Synergism

Fath's Cyber Ecosystems – Built from community assembly algorithm

B.D. Fath / Ecological Modelling 171 (2004) 329–337



Synergism always occurs (Fath 1998)
Increases with cycling
Decreases with model size

Fig. 4. Network synergism vs. cycling

Synergism	Positive utility exceeds negative utility giving rise to dominant positive relations.	$\frac{\sum +\text{utility}}{\lvert \sum -\text{utility} \rvert} > 1$

Conclusions

- In resource-flow systems, network synergism causes
  - Gain in positive utility due to indirect effects
  - Change in qualitative interactions types
  - …from less positive to more positive types
- Directly observed interactions types in nature are not the actually prevailing binary relationships expressed in long-term steady-state ecosystems

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Storage Utility Analysis

- Utility analysis shown in this lecture is based on the Flow analysis
- It is possible to run a utility analysis from a storage perspective
- Less used, beyond scope of this class
- Direct utility matrix calculation is different, but the remaining calculations are parallel.

$$\mathbf{DS} = [ds_{ij}] = (f_{ij} - f_{ji})/X_i$$

Suggested Activities

Suggested Utility Activities: enaR

- Load Oyster Reef Model
  - data(oyster)
- Perform Utility Analysis
  - U= enaUtility(oyster)
  - attributes(U)
- Compare results to those presented in this lecture
- Perform storage based Utility Analysis compare results to flow based Utility
  - enaUtility(oyster, type="storage")